

RIEMANN

<p>ExEuMetr: examples for →EuclM, Euclidean metric</p> <p>[OK] gives example</p>	<pre> 9: 8: 7: 6: 5: 4: 3: 2: 1: Ex: Euclidean Metric Spherical: $\{ \{ r \ \theta \ \varphi \}$ Cylindrical: $\{ \{ \varphi \ \rho \ z \}$ Circle S1: $\{ \{ \theta \ \varphi \} \}$ Sphere S2 $\theta, \varphi: \{ \{ \theta \ \varphi \}$ Sphere S2 $r, \varphi: \{ \{ r \ \varphi \}$ Torus T2: $\{ \{ \theta \ \varphi \} \}$ Sphere S3 $r, \theta, \varphi: \{ \{ r \ \theta \ \varphi \}$ Sphere S3 $\varphi, \theta, \varphi: \{ \{ \varphi \ \theta \ \varphi \}$ </pre>	<pre> 5: 4: 3: 2: 1: </pre> $\begin{Bmatrix} r \\ \theta \\ \varphi \end{Bmatrix}$ $\begin{Bmatrix} r \cdot \sin(\theta) \cdot \cos(\varphi) \\ r \cdot \sin(\theta) \cdot \sin(\varphi) \\ r \cdot \cos(\theta) \end{Bmatrix}$
<p>→EuMetr: Euclidean metric of coordinates or manifold, ex: sph. coordinates (0.5s) sphere S2 $\theta \ \varphi$ (0.2s)</p>	<pre> 3: 2: 1: </pre> $\begin{Bmatrix} \theta \\ \varphi \end{Bmatrix}$ $\begin{Bmatrix} r \cdot \sin(\theta) \cdot \cos(\varphi) \\ r \cdot \sin(\theta) \cdot \sin(\varphi) \\ r \cdot \cos(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \cdot \sin^2(\theta) \end{bmatrix}$	<pre> 5: 4: 3: 2: 1: </pre> $\begin{Bmatrix} \theta \\ \varphi \end{Bmatrix}$ $\begin{Bmatrix} r \cdot \sin(\theta) \cdot \cos(\varphi) \\ r \cdot \sin(\theta) \cdot \sin(\varphi) \\ r \cdot \cos(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \cdot \sin^2(\theta) \end{bmatrix}$
<p>→EuMetr: torus T2 (0.2s)</p> <p>sphere S3(0.5s)</p>	<pre> 4: 3: 2: 1: </pre> $\begin{Bmatrix} \theta \\ \varphi \end{Bmatrix}$ $\begin{Bmatrix} (R+r \cdot \cos(\theta)) \cdot \cos(\varphi) \\ (R+r \cdot \cos(\theta)) \cdot \sin(\varphi) \\ r \cdot \sin(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} r^2 & 0 & 0 \\ 0 & r^2 \cdot \cos^2(\theta) + 2r \cdot R \cdot \cos(\theta) + R^2 & 0 \\ 0 & 0 & r^2 \cdot \sin^2(\theta) \end{bmatrix}$	<pre> 4: 3: 2: 1: </pre> $\begin{Bmatrix} r \ \theta \ \varphi \end{Bmatrix}$ $\begin{Bmatrix} r \cdot \sin(\theta) \cdot \cos(\varphi) \\ r \cdot \sin(\theta) \cdot \sin(\varphi) \\ r \cdot \cos(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} R^2 & 0 & 0 \\ R^2 - r^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \cdot \sin^2(\theta) \end{bmatrix}$
<p>ExGMetr: examples for →GMetr, general metric</p> <p>spherical Minkowski (0.5s)</p>	<pre> 9: 8: 7: 6: 5: 4: 3: 2: 1: Ex: General Metric Spherical Minkowski: $\{ \{ r \ \theta \ \varphi \}$ Cylindrical Minkowski: $\{ \{ \varphi \ \rho \ z \}$ Hyperbolic H2: $\{ \{ r \ \theta \}$ Hyperbolic H3: $\{ \{ r \ \theta \}$ Spherical de Sitter: $\{ \{ r \ \theta \ \varphi \}$ Spher. Anti de Sitter: $\{ \{ r \ \theta \ \varphi \}$ </pre>	<pre> 5: 4: 3: 2: 1: </pre> $\begin{Bmatrix} r \cdot \sin(\theta) \cdot \cos(\varphi) \\ r \cdot \sin(\theta) \cdot \sin(\varphi) \\ r \cdot \cos(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<p>→GMetr: ex. hyperbolic H2 (0.1s)</p> <p>hyperbolic H3 (0.6s)</p>	<pre> 4: 3: 2: 1: </pre> $\begin{Bmatrix} r \ \theta \end{Bmatrix}$ $\begin{Bmatrix} r \cdot \cos(\theta) \\ r \cdot \sin(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} L^2 + r^2 & 0 \\ 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$	<pre> 2: 1: </pre> $\begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} L^2 & 0 & 0 \\ L^2 + r^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \cdot \sin^2(\theta) \end{bmatrix}$
<p>→GMetr: spherical de Sitter (3.5s)</p>	<pre> 5: 4: 3: 2: 1: </pre> $\begin{Bmatrix} t \ x \ \theta \ \varphi \end{Bmatrix}$ $\begin{Bmatrix} t \cdot \cosh(x) + t \cdot \sinh(x) \cdot \sin(\theta) \cdot \cos(\varphi) \\ t \cdot \sinh(x) \\ t \cdot \cosh(x) \cdot \sin(\theta) \cdot \sin(\varphi) \\ t \cdot \cosh(x) \cdot \cos(\theta) \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<pre> 3: 2: 1: </pre> $\begin{Bmatrix} t \ x \ \theta \ \varphi \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} -L^2 & 0 & 0 \\ L^2 + t^2 & 0 & 0 \\ 0 & t^2 & 0 \\ 0 & 0 & t^2 \cdot \sin^2(\theta) \end{bmatrix}$
<p>ExMetric: examples coordinates, metric</p>	<pre> 9: 8: 7: 6: 5: 4: 3: 2: 1: Ex: Metric +Conn, Geodes Sphere S2 $\theta, \varphi: \{ \{ \theta \ \varphi \}$ Sphere S2 $r, \varphi: \{ \{ r \ \varphi \}$ Torus T2: $\{ \{ \theta \ \varphi \} \}$ Hyperbolic H2 $r, \theta: \{ \{ r \ \theta \}$ Hyperbolic H2 $\varphi, \theta: \{ \{ \varphi \ \theta \}$ offdiagonal 2 dim: $\{ \{ r \ \theta \}$ Sphere S3: $\{ \{ r \ \theta \ \varphi \}$ Hyperbolic H3: $\{ \{ r \ \theta \ \varphi \}$ </pre>	<pre> 9: 8: 7: 6: 5: 4: 3: 2: 1: Ex: Metric +Conn, Geodes Sphere S3: $\{ \{ r \ \theta \ \varphi \}$ Hyperbolic H3: $\{ \{ r \ \theta \ \varphi \}$ H, B(t, r): $\{ \{ t \ r \ \theta \ \varphi \}$ Spherical symmetry 4d Schwarzschild: $\{ \{ t \ r \ \theta \ \varphi \}$ Eddington-Finkelstein Kruskal-Szekeres: $\{ \{ t \ r \ \theta \ \varphi \}$ FRM: $\{ \{ t \ r \ \theta \ \varphi \} \}$ </pre>
<p>M→Conn: metric to connection, (nonvanishing Christoffel symbols, note that $\Gamma^{\lambda}_{\nu\mu} = \Gamma^{\lambda}_{\mu\nu}$ is not shown and $\Gamma^{\lambda}_{\mu\nu} := \Gamma^{\lambda}_{\mu\nu}$) sphere S2 $r \ \varphi$ (0.6s)</p>	<pre> 5: 4: 3: 2: 1: </pre> $\begin{Bmatrix} r \ \varphi \end{Bmatrix}$ $g_{\mu\nu} = \begin{bmatrix} R^2 & 0 \\ R^2 - r^2 & 0 \\ 0 & r^2 \end{bmatrix}$	<pre> 2: 1: </pre> $\begin{Bmatrix} t \ r \end{Bmatrix}$ $\begin{Bmatrix} \Gamma^{\varphi}_{rr} = \frac{r}{R^2 - r^2} \\ \Gamma^{\varphi}_{r\varphi} = \left\{ -\frac{r \cdot R^2 - r^3}{R^2} \right\} \\ \Gamma^{\varphi}_{\varphi\varphi} = \frac{1}{r} \end{Bmatrix}$

C→Riemann: Schwarzschild connection (23s)	<pre> 5: :Prrr: '-(rS/(2*r*rS-2*r^2))' 6: :Prrt: '-((rS^2-r*rS)/(2*r^3))' 7: :Prrt: 'rS/(2*r*rS-2*r^2)' 8: :Prrt: 'rS-r' :Prrd: '(rS-r)*SIN' 9: :Prrt: '(0)^2' :Prrt: '1/r' :Prrd: '-(COS(0)*SIN(0))' :Prrd: '1/r' 10: :Prrd: 'COS(0)*SIN(0)' 11: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>	<pre> 4: :Rtrr: '-rS/(r^2*(rS-r))' 5: :Rtrt: '(rS-r)*SIN(r^4)' :Rtrt: '-rS/(2*r)' :Rtrt: '-((rS-r)*rS)/(2*r^4)' :Rtrd: '-(rS*SIN(0)^2)/(2*r)' :Rtrd: '-rS/(2*r)' 6: :Rtrd: 'rS/(2*r*(rS-r))' :Rtrd: '-(rS*SIN(0)^2)/(2*r)' :Rtrd: 'rS/(2*r*(rS-r))' 7: :Rtrd: 'rS*SIN(0)^2/r' :Rtrd: 'rS/r' 8: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>
C→Riemann: FLRW connection (30s)	<pre> 5: :Prrr: '-(a(t)*dia(t)/(r^2*k-1))' :Prrt: 'r^2*a(t)*dia(t)' 6: :Prrd: 'r^2*SIN(0)^2*a(t)*dia(t)' :Prrt: 'dia(t)/a(t)' :Prrt: '-(r*k/(r^2*k-1))' :Prrt: 'r^2*k-r' 7: :Prrd: 'r^2*k-r' :Prrd: 'r^2*k-r' :Prrd: 'dia(t)/a(t)' :Prrd: '1/r' 8: :Prrd: '-(COS(0)*SIN(0))' :Prrd: 'dia(t)/a(t)' :Prrd: '1/r' 9: :Prrd: 'COS(0)*SIN(0)' 10: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>	<pre> 5: :Rtrr: '-(a(t)*dia(t)/(r^2*k-1))' :Rtrt: '-dia(t)/a(t)' 6: :Rtrt: 'r^2*a(t)*dia(t)' :Rtrt: '-dia(t)/a(t)' :Rtrd: 'r^2*SIN(0)^2*a(t)*dia(t)' 7: :Rtrd: '-dia(t)/a(t)' :Rtrd: '(dia(t)^2*k)/(r^2*k-1)' :Rtrd: '(dia(t)^2*k)/(r^2*k-1)' :Rtrd: '(dia(t)^2*k)*r^2*SIN(0)^2' 8: :Rtrd: '-(dia(t)^2*k)/(r^2*k-1)' :Rtrd: '(dia(t)^2*k)*r^2*SIN(0)^2' :Rtrd: '(dia(t)^2*k)*r^2' 9: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>
C→Riemann: connection to Riemann tensor for offdiagonal metric. Ex: Vaidya metric	<pre> 3: 2: 1: </pre> $\begin{bmatrix} -\left(1-\frac{2H(u)}{r}\right) & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin(\theta)^2 \end{bmatrix}$ <p style="text-align: center;">Cu r 0 0</p> <p>H=Geo Geo=0 Geo=C ExC=R C=Ric C=Rsp</p>	<pre> 5: :Prrr: '-(H(u)/r^2)' :Prrt: 'r' 6: :Prrd: 'r*SIN(0)^2' :Prrt: '-(r^2*din(u)+(2*H(u)^2-r*H(u))/r^2)' :Prrt: 'H(u)/r^2' :Prrt: '2*H(u)-r' :Prrd: '2*SIN(0)^2*H(u)-r' 7: :Prrd: 'r*SIN(0)^2' :Prrt: '1/r' :Prrd: '-(COS(0)*SIN(0))' :Prrd: '1/r' 8: :Prrd: 'COS(0)*SIN(0)' 9: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>
C→Riemann: connection to Riemann tensor for offdiagonal metric. Ex: Vaidya metric (25s)	<pre> 1: </pre> $\begin{matrix} R_{\mu\nu\rho\sigma} \\ R_{\mu\rho\nu\sigma} \\ R_{\mu\nu\sigma\rho} \\ R_{\mu\sigma\nu\rho} \end{matrix}$ <p>H=Geo Geo=0 Geo=C ExC=R C=Ric C=Rsp</p>	<pre> 5: :Rrrr: '-(H(u)^2)/r^3' :Rrrr: '-(SIN(0)^2*H(u))/r' :Rrrr: '(r-2*H(u))*H(u)*2/r^4' :Rrrr: 'H(u)*2/r^3' :Rrrr: 'din(u)' :Rrrr: '-H(u)/r' :Rrrd: 'SIN(0)^2*din(u)' 6: :Rrrd: '-(SIN(0)^2*H(u))/r' :Rrrd: '-(r*H(u)-2*H(u)^2-r^2*din(u))/r^4' :Rrrd: '-H(u)/r^3' :Rrrd: '-H(u)/r^3' :Rrrd: 'SIN(0)^2*H(u)*2/r' :Rrrd: '-(r*H(u)-2*H(u)^2-r^2*din(u))/r^4' 7: +SKIP SKIP+ +DEL DEL+ DEL L INS+ </pre>
R→Ricci: Riemann to Ricci tensor, ex: Vaidya metric (2s)	<pre> 3: 2: 1: </pre> $\begin{bmatrix} 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin(\theta)^2 \end{bmatrix}$ <p style="text-align: center;">Cu r 0 0</p> $R_{\mu\nu} = \begin{bmatrix} -\frac{(din(u)+2)}{r^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ <p>C=Rsp ExC=R R=Ric R=Rsp R=Ric R=Ric</p>	<pre> 1: </pre> <p style="text-align: center;">R=0</p> <p>C=Rsp ExC=R R=Ric R=Rsp R=Ric R=Ric</p>
C→R ^λ _{μνρ} : calculate one component R ^λ _{μνρ} of the Riemann tensor Schwarzschild (0.3s) FLRW (0.7s) FriedmannRobertsonWalker	<pre> 6: 5: 4: 3: 2: 1: </pre> $\left\{ \begin{matrix} Prrr: \left(-\frac{rS}{2*r*rS-2*r^2} \right) & Prrt: \left(-\frac{rS^2}{2*r*rS-2*r^2} \right) \\ Rtrr: -\frac{rS}{r^2*(rS-r)} \end{matrix} \right.$ <p>H=Geo Geo=0 Geo=C ExC=R C=Ric C=Rsp</p>	<pre> 4: 3: 2: 1: </pre> $\left\{ \begin{matrix} Prrr: \left(-\frac{a(t)*dia(t)}{r^2*k-1} \right) & Prrt: (r^2*a(t)*dia(t)) \\ Rtrr: -\frac{a(t)*dia(t)}{r^2*k-1} \end{matrix} \right.$ <p>H=Geo Geo=0 Geo=C ExC=R C=Ric C=Rsp</p>
ExR→RE: examples for Riemann to Ricci, Einstein tensor	<pre> 9: 8: 7: 6: 5: 4: 3: 2: 1: </pre> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Riemann+Ricci, Einstein Sphere S2 0,0: 0 guv: Sphere S2 r,0: 0 guv: Schwarzschild: 0 guv: FRW: 0 guv: II '-1' 0 </div> <p style="text-align: center;">CANCEL OK</p>	<pre> 5: 4: 3: 2: 1: </pre> $\left\{ \begin{matrix} Rtrr: -\frac{rS}{r^2*(rS-r)} & Rtrt: \frac{(rS-r)*r}{r^4} \end{matrix} \right.$ $R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ <p>ExC=R R=Ric R=Rsp R=Ric R=Ric R=Ric</p>
R→Ricci: Riemann to Ricci Ex: Schwarzschild (1.5s)	<pre> 7: 6: 5: 4: 3: 2: 1: </pre> $\left\{ \begin{matrix} Rtrr: -\frac{a(t)*dia(t)}{r^2*k-1} & Rtrt: \frac{a(t)*dia(t)}{r^2*k-1} \end{matrix} \right.$ <p>ExC=R R=Ric R=Rsp R=Ric R=Ric R=Ric</p>	<pre> 3: 2: 1: </pre> $R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ <p>ExC=R R=Ric R=Rsp R=Ric R=Ric R=Ric</p>
R→Ricci: Ex: FLRW(2.1s)	<pre> 7: 6: 5: 4: 3: 2: 1: </pre> $\left\{ \begin{matrix} Rtrr: -\frac{a(t)*dia(t)}{r^2*k-1} & Rtrt: \frac{a(t)*dia(t)}{r^2*k-1} \end{matrix} \right.$ <p>ExC=R R=Ric R=Rsp R=Ric R=Ric R=Ric</p>	<pre> 3: 2: 1: </pre> $R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ <p>ExC=R R=Ric R=Rsp R=Ric R=Ric R=Ric</p>

MRi→R: metric and Ricci tensor to Ricci scalar ex: sphere S2 (0s)	<pre> 5: 4: 3: 3μν: [r^2 0 0 r^2.SIN(θ)^2] 2: Rμν: [1 0 0 SIN(θ)^2] 1: R: 2/r^2 </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>	<pre> 2: Γrrr: [-1/r^2] Γrrθ: [r^3-r] Γrrφ: [1/r] Γθrr: [r^2] Γφrr: [-1/(r+1)(r-1)] 1: </pre> <p>H→Geo Geo→D Geo→C ExC→R C→Ric C→Rμν</p>
M→Conn: offdiagonal Minkowski (2.6s)	<pre> 5: 4: 3: 2: 1: ξ t r u v3 [-1 0 0 0 0 1 0 0 0 0 1+u^2 -u+2.u 0 0 -u+2.u 1+u.u^2] </pre> <p>ExEuc→EucU ExHAn→HAnR ExHct→HctCon</p>	<pre> 4: 3: [-1 0 0 0 0 1 0 0 0 0 1+u^2 -u+2.u 0 0 -u+2.u 1+u.u^2] ξ t r u v3 2: 1: Γuuu: (u^2.u+2.u)/(u^2.u^2+u^2).u+4.u.u-(4.u^2-1) </pre> <p>ExEuc→EucU ExHAn→HAnR ExHct→HctCon</p>
C→Riemann: offdiagonal Minkowski (3.5s)	<pre> 1: Ruvuv: (1-4.u^2+4.u.u+(u^2+u^2.u^2)).(1+u^2).(4-q).u Ruvuv: (1-4.u^2+4.u.u+(u^2+u^2.u^2)).(2.u-q).(4-q).u Ruvuv: (1-4.u^2+4.u.u+(u^2+u^2.u^2)).(1+u^2).(4-q).u </pre> <p>H→Geo Geo→D Geo→C ExC→R C→Ric C→Rμν</p>	<pre> 1: Rμν: [0 0 0 0 0 0 (u^2+1).(q-4).u 0 0 ((u^2.u^2+u^2).u+4.u.u-(4.u^2-1)) 0 0 -(u-2.u).(q-4).u 0 0 ((u^2.u^2+u^2).u+4.u.u-(4.u^2-1))] </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>
R→Ricci: (1.5s)		
MRi→R: metric and Ricci tensor to Ricci scalar (2s). For a=4 R=0.	<pre> 7: 6: 5: 4: 3: 2: 1: R: (a-4).u.2/((u^2.u^2+u^2).u+4.u.u-(4.u^2-1))^2 </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>	<pre> 2: 1: 3μν: [0 0 0 -1 0 0 0 a(t)^2/(1-kr^2) 0 0 0 a(t)^2.r^2 0 0 0 a(t)^2.r^2] </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>
FLRW metric		
FLRW Ricci tensor	<pre> 2: 1: Rμν: [0 0 0 a(t)^2.r^2 -3.dida(t)/a(t) 0 0 0 -a(t).dida(t)/r^2 0 0 0] </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>	<pre> 2: 1: Gμν: [3.dida(t)^2+3.k/a(t)^2 0 0 2.a(t).dida(t)/r^2 0 0] </pre> <p>ExR→R R→Ric R→Rμν RLi→R RLi→E TrIHQ</p>
MRi→G _{μν} : metric and Ricci tensor to Einstein tensor Ex: FRW (3.5s)		
ExGT→Eeq: example Einstein G and energy mom. tensor T to Einstein equations ex: FLRW	<pre> 2: 1: Tμν: [0 0 0 p.a(t)^2/(1-kr^2) 0 0 0 p.a(t)^2.r^2 0 0 0 p.a(t)^2] </pre> <p>ExGT→GT→Eeq HelpR</p>	<pre> 1: [1.dida(t)^2+k/a(t)^2 2.a(t).dida(t)+dida(t)/r^2.k-1 -((2.a(t).dida(t)+dida(t)^2+k).r^2) -((2.a(t).dida(t)+dida(t)^2+k).r^2)] </pre> <p>ExGT→GT→Eeq HelpR PPAR</p>
GT→Eeq (0.5s)		
1. equation (before and after simplifying)	<pre> 6: 5: 4: 3: 2: (dida(t)^2+k).3=k.p(t) a(t)^2 1: dida(t)^2+k=k.p(t).a(t)^2/3 </pre> <p>List ExEuc→EucU ExHAn→HAnR ExHct→HctCon</p>	<pre> 7: 6: 5: 4: 3: 2: 2.a(t).dida(t)+dida(t)^2+k=k.p(t)/r^2 1: 2.a(t).dida(t)+dida(t)^2+k=k.p(t)/r^2 </pre> <p>List ExEuc→EucU ExHAn→HAnR ExHct→HctCon</p>
M→Ricci: diagonal metric to Ricci tensor ex: sphere S2 (3s)	<pre> 3: 3μν: [1/r^2 0 0 r^2] { r θ φ } 2: Rμν: [-1/(r+1)(r-1) 0 0 r^2] </pre> <p>ExGT→GT→Eeq HLi→R HLi→Ric ExLi→TrIHQ</p>	<pre> 4: 3: 3μν: [r^2 0 0 (R+r.COS(θ))^2] { θ φ } 2: Rμν: [r.COS(θ)/(r.COS(θ)+R) 0 0 (r.COS(θ)+R).COT(θ)/r] </pre> <p>ExGT→GT→Eeq HLi→R HLi→Ric ExLi→TrIHQ</p>
ex: torus T2 (4s)		

HelpRIEMANN: help to programs	<pre> M→Riemann {x^2} [[g_{μν}]] + [[g_{μν}]] {x^2} {R^{μνρσ}} Metric + Riemann tensor M→Ricci {x^2} [[g_{μν}]] + [[g_{μν}]] {x^2} [[R_{μν}]] Metric + Ricci tensor M→R {x^2} [[g_{μν}]] + R Metric + Ricci scalar dM→Kret {x^2} [[g_{μν}]] + R diagonal Metric to R = R_{μνρσ} R^{μνρσ} 4 Kretschmann scalar +SKIP[SKIP+] +DEL DEL+ DEL L INS = </pre>	<pre> dM→T_{μν} [[g_{μν}]] + [[g_{μν}]] [[T_{μν}]] diagonal Metric in 4 dim to energy momentum tensor of fluid in comoving coordinates (u⁰)²=g₀₀ uⁱ=0, ex:FLRW d+der 'd2dg_{μν}(t,r)' + '30(ar(g_{μν}(t,r)))' works for d1..d3 TriHyp [1],[2] + [1]' simplify terms with SIN(θ),COS(θ) Collect [1],[2] + [1]',[2]' +SKIP[SKIP+] +DEL DEL+ DEL L INS = </pre>
HelpRIEMANN: help to programs	<pre> Fdistrib [1],[2] + [1]',[2]' FDISTRIB for Matrix,list Eval [1],[2] + [1]',[2]' EVAL for Matrix,list LinT [2],[1] + .. linearise terms in list,Matrix trig. hyperbolic like SIN(θ) .. not linearised Ex: Goedel Subst [1],[2] {'v1=v1'..} substitute variables performs d+der +SKIP[SKIP+] +DEL DEL+ DEL L INS = </pre>	<pre> Match [1],[2] {t1,t2} + [1]',[2]' Ex:Eddington Finkelstein Riemann tensor substitute {a^2} 13 Powsimp [1],[2] + [1]',[2]' power simplify terms like 'RR00T(3,8)*JX' Mxds2 {x^2} g_{μν}: [1] + {x^2} ds^2 ds^2=g_{μν}dx^μdx^ν Metric Matrix + line element d2d1+d1d2 [1] + [1]' replace d2dg(t,r) + d1d2g(t,r) +SKIP[SKIP+] +DEL DEL+ DEL L INS = </pre>

The software RIEMANN provides tools for calculating metric, connection, geodesic equations, Riemann and Ricci tensor and Einstein equations. The programs should be used on the HP50 emulator on a computer, smart phone or tablet. The software requires the MPC font for correct display of tensor indices.

How do the programs work?

M→Conn: The direct variation of $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ yields the geodesic equations

$\frac{d^2 x^\lambda}{ds^2} + \Gamma_{\mu\nu}^\lambda \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = 0$ with M→Geodes, from which the non vanishing Christoffel symbols of second kind $\Gamma_{\mu\nu}^\lambda$ are obtained as a list with Geo→Conn.

(Ex: FLRW M→Conn 12s/3.5s on a slow/fast smart phone)

C→Riemann: calculates all components of the Riemann tensor via brute force with $C \rightarrow Riemann$ and $C \rightarrow R_{\lambda\mu\nu}^\rho$ from the non vanishing Christoffel symbols. Only non zero components are given (Ex: FLRW 30s/9.5s). For more complicated metrics as Kerr one should use one of the many computer programs like GRtensor, which employ Cartan's calculus.

Several algebraic tools help to simplify terms in lists or matrices: TriHyp, Collect, Fdistrib, Eval, LinT. With Subst and Match you can substitute functions or perform matches.

Check all examples in RIEMANN and the over 30 metrics in RIEM2 with the programs.

Some important books on the subject are (among many others):

- A. Zee, Einstein Gravity in a Nutshell
- L. Ryder, Introduction to General Relativity
- M. Nakahara, Geometry, Topology and Physics